CEGE 3201 application: addressing the liberal education requirements

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Contents

1 Summary of CEGE 3201

CEGE 3201: Introduction to Transportation Engineering is designed to be the first course students take on designing and modeling transportation systems. Transportation encompasses several major areas of technology, including vehicles themselves, road infrastructure, and public transit systems. All students, and most Minnesota residests, have used transportation systems in Minnesota, and probably have their share of concerns, criticisms, or desired design changes to them. For instance, a common criticism is that the bus system here has much higher travel times than driving, and does not provide as much connectivity as desired. However, most users of the transportation system have little understanding of why our transportation systems are designed and operated the way they are, how traffic congestion forms, or how individual traveler behaviors are predicted to affect the overall system. Transportation engineers have spent decades working on these systems, and this class provides an overview of these engineering principles.

CEGE 3201 is a required course for students studying towards a bachelor's degree in civil engineering. Many civil engineering problems interact with transportation systems, such as the design of bridges for vehicles or pedestrians; the environmental impacts of vehicle traffic (including greenhouse gas emissions and other pollutants), interactions with water resource systems like drainage, etc. For many of these students, CEGE 3201 is the only class they take on transportation engineering. CEGE 3201 provides a different, multidisciplinary perspective on urban infrastructure for such students.

CEGE 3201 is also open to non-majors; for instance, students from computer science and industrial engineering who are interested in transportation systems take CEGE 3201. Although some of the stated co-requisites are CEGE classes, these co-requisites consist of fundamental skills can be replaced by equivalent classes from other departments. CEGE 3201 has the following prerequisites:

- PHYS 1301: kinematic physics
- CEGE 3101, or any course on computer programming
- CEGE 3102, or any probability and statistics course

2 Criteria for all LE courses

I will discuss how CEGE 3201 meets the requirements of liberal education courses, addressing each requirement separately.

- Explicitly help students understand what liberal education is, how the content and the substance of this course enhance a liberal education, and what this means for them as students and as citizens;
- Meet one or more of the Student Learning Outcomes (SLOs). In the syllabus you submit, specify which of the SLO(s) that the course meets, how it addresses the outcome(s), and how the learning that is associated with the outcome(s) will be assessed;
- Be offered on a regular schedule;

- Be taught by regular faculty or under exceptional circumstances by instructors on continuing appointments. Departments proposing instructors other than regular faculty must provide documentation of how such instructors will be trained and supervised to ensure consistency and continuity in courses;
- Be at least 3 credits (or at least 4 credits for biological or physical sciences, which must include a lab or field experience component).

2.1 Explicitly help students understand what liberal education is

Most students have some preconceived notions of transportation systems based on years of navigating them. Many have identified issues with transportation: traffic congestion; inefficient public transit, etc. Part of developing a liberal education is approaching existing aspects of society with a scientific mind. CEGE 3201 provides a rigorous perspective on transportation systems founded on engineering design principles and scientific study of traveler behaviors. For example, traffic congestion is studied in terms of route choices leading to a high concentration of vehicles on certain roads, as well as traffic flow theory for modeling the behavior of traffic congestion and drivers in such traffic. These topics are treated with engineering rigor and mathematical modeling. By studying such engineering, students gain a deeper understanding of transportation systems, and are more prepared to understand their issues and find informed solutions. This helps students understand part of a liberal education — a scientific study of a critical system to society that most people interact with but have little understanding of. After completing CEGE 3201, students are prepared for informed discussions about issues with our transportation systems and potential solutions. The content and substance of this course enhance a liberal education by providing that rigorous study of transportation systems. As citizens, students will be able to make informed decisions about using, navigating, and improving transportation systems, and participate appropriately in venues such as voting on electric vehicle tax incentives, infrastructure funding, etc.

2.2 Student Learning Outcomes (SLOs)

CEGE 3201 addresses two SLOs:

- Problem solving
- Critical thinking

Section 4.5 of the syllabus discusses how CEGE 3201 meets these SLOs and how they will be assessed.

2.2.1 Problem solving

CEGE 3201 is mostly about solving engineering problems around modeling transportation systems to understand and predict their behavior or designing transportation systems to improve their behavior. I have included two sample homework problems that students are able to solve during past iterations of CEGE 3201 in Appendices A.1 and A.2. The user equilibrium problem in Appendix A.1 requires students to set up a system of linear equations describing user equilibrium behavior. They have to figure out how to apply conceptual behaviors to a specific problem. The problem in Appendix A.2 requires students to apply queueing theory equations to an airport security line problem. Although students know certain equations to calculate average waiting times and queue lengths for queueing theory, it is not immediately obvious how to use them to address the given problem.

2.2.2 Critical thinking

In-class and homework problems also require students to critically evaluate solution methods for engineering problems. Transportation technology is currently being revolutionized by automated vehicles, and many of these problems center around how transportation engineering design principles should be modified for automated vehicles. I have provided two example homework problems and their solutions in Appendices A.3 and A.4. The problem in Appendix A.3 requires students to analyze current traffic signal design, which is calibrated for human drivers, and propose how these may or may not be changed due to vehicle automation. There are many correct answers, but answers have to be reasonably justified. The problem in Appendix A.4 similarly asks students to propose changes in the fundamental flow-density relationship describing traffic flow behavior.

2.3 Be offered on a regular schedule

CEGE 3201 is taught every fall and spring semester.

2.4 Be taught by regular faculty

CEGE 3201 is only taught by tenured or tenure-track faculty in the Civil, Environmental, and Geo-Engineering department.

2.5 Be at least 3 credits

CEGE 3201 is 3 credits, and includes 2hrs of lecture and a 1hr lab of transportation data analysis per week.

3 Criteria for all theme courses

I will discuss how CEGE 3201 meets each requirement of theme courses, addressing each requirement separately.

- thinking ethically about important challenges facing our society and world;
- reflecting on the shared sense of responsibility required to build and maintain community;
- connecting knowledge and practice;
- fostering a stronger sense of our roles as historical agents.

3.1 Thinking ethically about important challenges facing our society and world

Efficient transportation is itself an important challenge for society. We rely on transportation systems to commute to work each day, and transportation delivers food to grocery stores for us to purchase. There are many ethical questions around the design of transportation systems. For example, should we try to minimize the total congestion (which might result in more congestion for certain areas) or focus on providing equal access to different minorities? Should we design public transit to serve demand or emphasize it to reduce private vehicle traffic and the corresponding traffic congestion and greenhouse gas emissions? Tradeoffs like this are inherent in engineering problems which affect all people. The homework problem in Appendix A.5 asks students to consider the tradeoff between ridesharing and public transit in the public transit design. Students must consider how changing the frequency of the light rail train reduces the capacity to match demand, but also affects people's behavior and decreases ridership. They must therefore confront the choice between designing a system that encourages public transit use versus designing a system for maximum efficiency.

3.2 Reflecting on the shared sense of responsibility required to build and maintain community

In one sense, transportation systems are designed by the engineers, companies, and government agencies responsible for designing them. However, traveler behaviors collectively demand a transportation system that works for them. Much of transportation engineering is creaeting aggregate models describing the behavior of travelers. In that sense, travelers have a shared responsibility to build the transportation system that they desire. If a significant proportion of travelers shift towards public transit, engineers will design the public transit system accordingly and will have the corresponding fare revenue to use in the design. One example of a CEGE 3201 class activity which asks students to reflect on this shared sense of responsibiility is relating user equilibrium route choice to the prisoner's dilemma from game theory. Slides for this exercise are shown in Appendix A.8. The prisoner's dilemma is a scenario in which individuals are incentivized to behave in their own best interests; there is an individual advantage gained by trying to minimize their individual penalties. However, because every individual behaves similarly, the collective penalties are larger. A similar situation happens with drivers in how individual drivers seek to minimize their own travel time, but in doing so cause traffic congestion to be worse for everyone. This shows a shared responsibility among the participants in transportation systems; by reducing selfish motivations, the overall system can be improved.

3.3 Connecting knowledge and practice

Transportation engineering in practice requires the assistance of computer tools to study the transportation systems of large cities. CEGE 3201 explains some theory of how transportation models are developed, but also aims to connect that knowledge with how such models are used in practice. For example, after developing the concept of user equilibrium, students are given an Excel homework problem to find user equilibrium using a numerical method similar to those used in practice. This helps students understand how the knowledge is applied in practice and what to look for when using computer tools to solve this problem. The homework problem is presented in Appendix A.6. Students are given a simple network with realistic parameters that could be used for real cities. This problem is not easy to solve through equations; instead, students must use the method of successive averages, which is an algorithm used to solve user equilibrium problems on much larger networks. This helps students connect the theory of what user equilibrium is to the practical aspects of how to find it and the challenges of finding an exact solution. In fact, an exact solution requires infinite iterations to attain, but the solution will get progressively better with each iteration of the method of successive averages. Students will learn how to determine the accuracy of their solution.

3.4 Fostering a stronger sense of our roles as historical agents

Design choices around transportation infrastructure and systems have major impacts on society. For instance, highways have sometimes been built through neighborhoods of certain races or economic backgrounds rather than others. Public transit access is better for certain areas of Minnesota than others. Many of these decisions continue to affect us today. Hard infrastructure like roads and highways cannot easily be moved, and transportation connectivity has affected the development of different areas of the city. Understanding how transportation systems are designed will help students understand the options that transportation engineers had, and how they decided among those options. For example, refer to the problem on designing the TSA Pre✓security lines in Appendix A.2. The numbers are designed so that the only two solutions to the problem are to have either 1 or 2 TSA Pre✓lines. With 0 TSA Pre✓lines, the capacity is insufficient for the demand. With only 1 TSA Pre✓line, the TSA Pre✓and regular lines have roughly equal waiting times. This would seem to be fair. However, in such a scenario, passengers would no longer pay for TSA Pre✓, which would eventually revert to the 0 TSA Pre✓line scenario in which the capacity is insufficient. Therefore, students can understand that the seemingly unfair security lane allocation is motivated by other specifics of the problem structure. This is an example of how previous engineering decisions may seem obviously wrong now, but can be better understood through a rigorous engineering analysis.

4 Criteria for the Technology and Society theme

Although nowadays we take transportation for granted, 100 years ago the lack of automobiles affected city life and urban layouts. Transportation is therefore a technology that has had a profound impact on society, and emerging developments in vehicle automation are likely to revolutionize how we interact with transportation systems. Becoming informed about transportation system design is a necessary step for students to become thoughtful consumers of transportation technologies.

CEGE 3201 considers the impact of technology on society by creating models of traveler behaviors. These models require connecting a conceptual understanding of people's choices, such as where they choose to live and work, with transportation options. For example, consider the homework problem in Appendix A.7. Students must predict how changes in travel times due to technological changes in transportation would affect people's choices. The problem proposes some changes in travel times due to vehicle automation, which could affect where people choose to live and work. Students must adapt engineering methods to calculate the new travel choices.

CEGE 3201 also discusses how society's responses to transportation shapes its use, and develops mathematical models of that behavior. For instance, consider the slides in Appendix A.8, where selfish user behaviors negatively affect the outcome of the system. When driving, most of us try to choose routes that will minimize our travel time. However, such routes will create more congestion, causing all drivers to experience higher travel times. This is an example of how the responses of society shape the use of transportation technology and the design of transportation systems. Students are asked to understand such behaviors conceptually and create models of them to predict their impacts.

Another related problem is presented in Appendix A.5, showing how users respond to transportation systems; the frequency of light rail trains affects the capacity as well as ridership, which is a traveler choice. As engineers try to adjust the frequency of trains for efficiency by matching the passenger capacity to the demand, their adjustment itself affects demand because lower frequencies cause higher waiting times for travelers. Consequently, CEGE 3201 students are introduced to many perspectives on the use of technology. These include the perspective of the engineers who plan and design transportation systems; these engineers are mostly concerned with the efficient operation of transportation systems. Students also study and model the perspective of individual users in choosing which aspects of the transportation system to use, such as which roads to drive on or which modes (driving or public transit) to use in their commute.

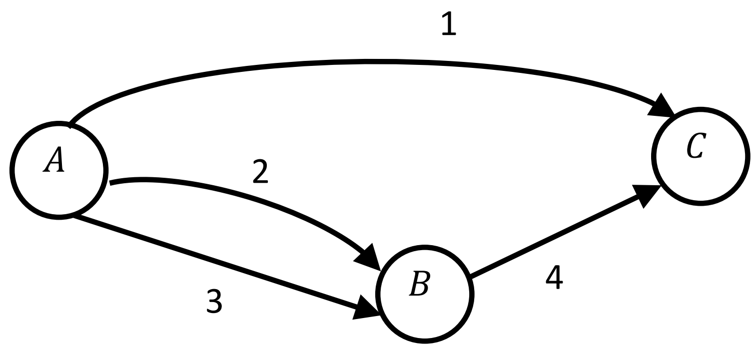
5 Syllabus requirements

- Section 2 of the syllabus describes how CEGE 3201 is a liberal education course.
- Section 2 also describes how CEGE 3201 fulfills the Technology and Society theme.
- Section 3 describes the course format and in-class activities for active learning. Section 4 describes the graded activities.
- Section 6 gives a tentative schedule of topics. Each topic has one or more associated readings, which will be posted on Canvas on the appropriate day. The textbook is given in Section 3.2 of the syllabus.

A Sample problems from class

A.1 Finding user equilibrium

Consider the network shown below:



Link performance functions for each link are as follows:

$$t_1 = 5 + x_1 \qquad t_2(x_2) = 1 + 2x_2 \qquad t_3(x_3) = 10 + x_3 \qquad t_4(x_4) = 3 + 2x_4$$

where x is the link flow **in thousands of vehicles per hour** and t is in units of minutes.

Suppose the demand from A to C is 8,000 vph. Using a system of linear equations, find user equilibrium. Specify the **link** flows and travel times.

Solutions Start by setting the path travel times to be equal. Define path 1 as [1], path 2 as $[2, 4]$, and path 3 as $[3, 4]$. Define h_1, h_2, and h_3 as the path flows in thousands of vehicles per hour on paths 1, 2, and 3, respectively. Link flows are

$$\begin{aligned} x_1 &= h_1 \\ x_2 &= h_2 \\ x_3 &= h_3 \\ x_4 &= h_2 + h_3 \end{aligned}$$

Link travel times are

$$\begin{aligned} t_1 &= 5 + h_1 \\ t_2 &= 1 + 2h_2 \\ t_3 &= 10 + 3h_3 \\ t_4 &= 3 + 2(h_2 + h_3) \end{aligned}$$

Path travel times are

$$\begin{aligned} T_1 &= 5 + h_1 \\ T_2 &= 1 + 2h_2 + 3 + 2(h_2 + h_3) \\ T_3 &= 10 + h_3 + 3 + 2(h_2 + h_3) \end{aligned}$$

Set these equal at user equilibrium. From $T_1 = T_2$, obtain

$$5 + h_1 = 1 + 2h_2 + 3 + 2(h_2 + h_3)$$

From $T_1 = T_3$, obtain

$$5 + h_1 = 10 + h_3 + 3 + 2(h_2 + h_3)$$

The last equation relates the demand:

$$h_1 + h_2 + h_3 = 8$$

The solution to this system of equations is

$$h_1 = \frac{79}{11}$$
$$h_2 = \frac{36}{11}$$
$$h_3 = -\frac{27}{11}$$

Since $h_3 < 0$, set $h_3 = 0$ and resolve with the remaining two paths (set $h_3 = 0$). From $T_1 = T_2$, obtain

$$10 + 2h_1 = 3 + 4h_2 + 2 + 2h_2$$

and the second equation is

$$h_1 + h_2 = 8$$

The solution is

$$h_1 = \frac{31}{5}$$
$$h_2 = \frac{9}{5}$$
$$h_3 = 0$$

Link flows are

$$x_1 = h_1 = \frac{31}{5}\text{vph}$$
$$x_2 = h_2 = \frac{9}{5}\text{vph}$$
$$x_3 = h_3 = 0\text{vph}$$
$$x_4 = h_2 + h_3 = \frac{9}{5}\text{vph}$$

Link travel times are

$$t_1 = 5 + h_1 = 11.2\text{min}$$
$$t_2 = 1 + 2h_2 = 4.5\text{min}$$
$$t_3 = 10 + 3h_3 = 10\text{min}$$
$$t_4 = 3 + 2(h_2 + h_3) = 7.6\text{min}$$

Path travel times are

$$
\begin{aligned}
T_1 &= 5 + h_1 = 11.2\text{min} \\
T_2 &= 1 + 2h_2 + 3 + 2(h_2 + h_3) = 11.2\text{min} \\
T_3 &= 10 + h_3 + 3 + 2(h_2 + h_3) = 17.6\text{min}
\end{aligned}
$$

This is user equilibrium.

You may use other variables in place of h_1, h_2, and h_3 (e.g. x, y, and z) to simplify the writing as long as they are clearly defined.

A.2 Airport queue modeling and checkpoint design

After checking in at the airport, passengers walk to and join the security checkpoint line. Passengers arrive at the rate of 2.82 per minute. 30% of these passengers have purchased TSA Pre✓ (expedited security check).

TSA has 12 X-ray scanners total. Each scanner provides either TSA Pre✓ or standard service. Passengers who have purchased TSA Pre✓ will choose TSA Pre✓ lines unless none are available. Passengers who have not purchased TSA Pre✓ must use the standard lines.

Standard screening require 5 minutes on average. TSA Pre✓ screening requires only 1 minute on average. If no lines are designated as TSA Pre✓, all passengers (including those with TSA Pre✓) must endure standard 5min screening.

(a) Decide how many of the 12 scanners are allocated to standard screening and to TSA Pre✓ screening. Justify your choice. Does your choice minimize queue length, waiting time, or some other objective?

(b) TSA Pre✓ costs \$85 per 5 years. Those 30% of passengers with TSA Pre✓ will not renew their TSA Pre✓ subscription unless it saves them significant time. Does that affect the choice you made in part (a)?

Solutions The queueing model is $M/M/n$ because passenger arrivals are random (different walking speeds from the check-in counter, among other things); passenger departures are random (different numbers of bags to send through the scanner); and there are multiple scanners.

The equations for $M/M/n$ queueing are given in Section 5.5.5 of Mannering. I entered them in Excel to compare the allocation of scanners between standard screening and TSA Pre✓. Notice that if all 12 scanners are on standard screening, all passengers (including TSA Pre✓ passengers) must endure standard screening.

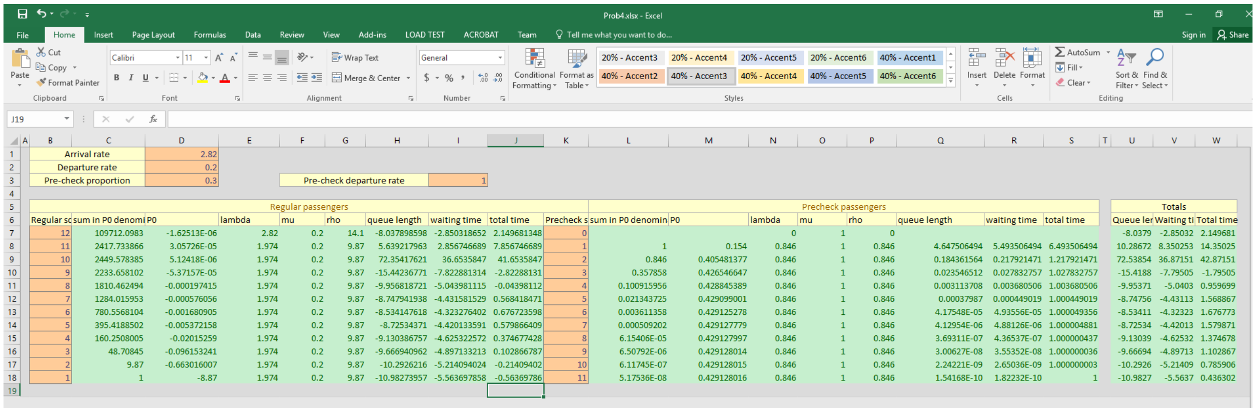
You can reduce the number of calculations by observing that when at least 1 TSA Pre✓ scanner is active, ρ for the non-TSA Pre✓ queue is

$$\rho = \frac{\lambda}{\mu} = \frac{2.82 \times 30\%}{0.2} = 9.87$$

which requires either 10–12 scanners open. When 0 TSA Pre✓ scanners are active, ρ for the non-TSA Pre✓ queue is

$$\rho = \frac{\lambda}{\mu} = \frac{2.82}{0.2} = 14.1$$

which cannot be served with 12 scanners. (When 0 TSA Pre✓ scanners are active, $\lambda = 2.82$ because all passengers use the same standard line.) So there are only two possibilities, 11 standard and 1 TSAPre✓, or 10 standard and 2 TSA Pre✓.



Arrival rate	2.82			
Departure rate	0.2			
Pre-check proportion	0.3		Pre-check departure rate	1

Regular passengers								
Regular sc	sum in P0 denomi	P0	lambda	mu	rho	queue length	waiting time	total time
12	109712.0983	-1.62513E-06	2.82	0.2	14.1	-8.037898598	-2.850318652	2.149681348
11	2417.733866	3.05726E-05	1.974	0.2	9.87	5.639217963	2.856746689	7.856746689
10	2449.578385	5.12418E-06	1.974	0.2	9.87	72.35417621	36.6535847	41.6535847
9	2233.658102	-5.37157E-05	1.974	0.2	9.87	-15.44236771	-7.822881314	-2.82288131
8	1810.462494	-0.000197415	1.974	0.2	9.87	-9.956818721	-5.043981115	-0.04398112
7	1284.015953	-0.000576056	1.974	0.2	9.87	-8.747941938	-4.431581529	0.568418471
6	780.5568104	-0.001680905	1.974	0.2	9.87	-8.534147618	-4.323276402	0.676723598
5	395.4188502	-0.005372158	1.974	0.2	9.87	-8.72534371	-4.420133591	0.579866409
4	160.2508005	-0.02015259	1.974	0.2	9.87	-9.130386757	-4.625322572	0.374677428
3	48.70845	-0.096153241	1.974	0.2	9.87	-9.666940962	-4.897133213	0.102866787
2	9.87	-0.663016007	1.974	0.2	9.87	-10.2926216	-5.214094024	-0.21409402
1	1	-8.87	1.974	0.2	9.87	-10.98273957	-5.563697858	-0.56369786

	Precheck passengers								Totals		
Precheck s	sum in P0 denomin	P0	lambda	mu	rho	queue length	waiting time	total time	Queue ler	Waiting ti	Total time
0			0	1	0				-8.0379	-2.85032	2.149681
1	1	0.154	0.846	1	0.846	4.647506494	5.493506494	6.493506494	10.28672	8.350253	14.35025
2	0.846	0.405481377	0.846	1	0.846	0.184361564	0.217921471	1.217921471	72.53854	36.87151	42.87151
3	0.357858	0.426546647	0.846	1	0.846	0.023546512	0.027832757	1.027832757	-15.4188	-7.79505	-1.79505
4	0.100915956	0.428845389	0.846	1	0.846	0.003113708	0.003680506	1.003680506	-9.95371	-5.0403	0.959699
5	0.021343725	0.429099001	0.846	1	0.846	0.00037987	0.000449019	1.000449019	-8.74756	-4.43113	1.568867
6	0.003611358	0.429125278	0.846	1	0.846	4.17548E-05	4.93556E-05	1.000049356	-8.53411	-4.32323	1.676773
7	0.000509202	0.429127779	0.846	1	0.846	4.12954E-06	4.88126E-06	1.000004881	-8.72534	-4.42013	1.579871
8	6.15406E-05	0.429127997	0.846	1	0.846	3.69311E-07	4.36537E-07	1.000000437	-9.13039	-4.62532	1.374678
9	6.50792E-06	0.429128014	0.846	1	0.846	3.00627E-08	3.55352E-08	1.000000036	-9.66694	-4.89713	1.102867
10	6.11745E-07	0.429128015	0.846	1	0.846	2.24221E-09	2.65036E-09	1.000000003	-10.2926	-5.21409	0.785906
11	5.17536E-08	0.429128016	0.846	1	0.846	1.54168E-10	1.82232E-10	1	-10.9827	-5.5637	0.436302

From the Excel spreadsheet, you can see that there are only two solutions in which all passengers are served. The solution that minimizes queue length, waiting time, and total time is to have 11 scanners for standard screening and 1 scanner for TSA Pre✓. The alternative solution is to have 10 scanners for standard screening and 2 scanners for TSA Pre✓.

Notice that if there are 11 standard scanners and 1 TSA Pre✓, then the difference in total time between standard and TSA Pre✓ screening is around 1.5 minutes. In fact, people in the standard line will wait on average 2.8min, but passengers in the TSA Pre✓ line will wait on average 5.5min! Not many people will pay \$85 to save 1.5min at the airport and spend an extra 2.7min in line. However, if no one uses TSA Pre✓, then we have the situation of 12 standard scanners and 2.82 passengers per hour need standard screening. There are not enough scanners to keep up (in Excel, P_0 is negative for 12 standard scanners). Therefore, we need to encourage some travelers to buy TSA Pre✓. The allocation that incentivizes TSA Pre✓ is 10 standard scanners and 2 TSA Pre✓ scanners.

A.3 Modifying traffic signals for automated vehicles

What changes would you make to the following timings for autonomous vehicles? Explain your reasoning for every change you make, and everything you do not change.

(a) The duration of the clearance (all-red) interval.

(b) The duration of the change (yellow) interval.

(c) Startup lost time at the start of each phase.

Solutions

- The clearance interval is based on kinematics — the time to clear the intersection — and therefore would not change.
- For the change interval, the reaction time might be much less for autonomous vehicles. The stopping distance component would be the same.
- Alternatively, you could completely remove change intervals if the light duration is communicated to autonomous vehicles wirelessly instead of through traffic lights.
- The startup lost time could be reduced either through smaller computer reaction times, or by communicating the start of the green light to vehicles in advance.

A.4 Modifying the standard flow-density relationship for automated vehicles

Consider the triangular flow-density relationship:

$$q = \min\{u_{\mathrm{f}}k, -w(k - k_{\mathrm{j}})\}$$

Suppose we want to modify this model for autonomous vehicles. What changes, if any, would you make to the following parameters? Briefly discuss each parameter separately.

(a) Free-flow speed u_{f}

(b) Congested wave speed w

(c) Jam density k_{j}

How will these changes affect road capacity?

Solutions

(a) Speed limits (free flow speeds) may increase because computers can react faster than humans, reducing the stopping sight distance.

(b) Reductions in following headway would also increase the congested wave speed.

(c) The jam density would remain the same because vehicle length would not change.

(d) Increases in the free flow speed or the congested wave speed will increase road capacity.

A.5 Designing light rail frequency for capacity and modal split

Business travelers (who do not rent a car) have two main options to travel from the airport to downtown Minneapolis: the blue LRT line and ridesharing. MetroTransit has estimated the following model for mode choices.

Mode	Alternative specific constant	Waiting time	In-vehicle travel time	Cost
Ridesharing	0	7 min	14 min	\$19.4
LRT	17.5	TBD	27 min	\$2.5

The coefficient for all types of travel time (in min) is 3.5. The coefficient for cost (in \$) is 2.323. The logit dispersion is 0.04.

Light rail trains carry 150 passengers each. The total demand using either LRT or ridesharing is 3000 passengers per hour. Travelers will choose either LRT or ridesharing based on the utility to them, and that utility depends on the waiting time.

The minimum headway for LRT is 5 minutes. What headway should the LRT be scheduled at so that LRT capacity is equal to LRT demand? What is the LRT demand and capacity at that headway?

Solutions The appropriate waiting time model is $\frac{\text{headway}}{2}$ because traveler arrivals are based on flight arrivals, which have no correlation to LRT arrivals.

- The disutility for ridesharing is $0 + 3.5 \times (7 + 14) + 2.323 \times 29.4 = 118.57$
- The disutility for LRT is $17.5 + 3.5 \times \left(\frac{h}{2} + 27\right) + 2.323 \times 2.5$.
- Use a logit model to find the probability a traveler chooses LRT. The number of travelers taking LRT is

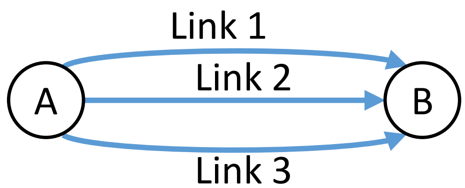
$$3000 \left(1 - \frac{\exp -0.04 \times 124.87}{\exp -0.04 \times 124.87 + \exp -0.04 \times \left(17.5 + 3.5\left(\frac{h}{2} + 27\right) + 2.323 \times 2.5\right)} \right)$$

- The capacity of the light rail mode is $\frac{150\text{pass/hr}}{h/60}$ where h is in minutes.

I used Excel to find where these two functions are equal, and came up with 8.149 minutes. Graphing calculators should also be able to estimate intercepts. At a headway of 8.149 minutes, LRT demand is 1104, and capacity is 1104.

A.6 Finding user equilibrium using the method of successive averages

Consider the network shown below:



The demand from A to B is 8000 vehicles per hour.

Each link is also a path. Suppose the travel times for link a follow the Bureau of Public Roads (BPR) function:

$$T_a(x_a) = \bar{t}_a \left(1 + \alpha_a \left(\frac{h_a}{Q_a}\right)^{\beta_a}\right)$$

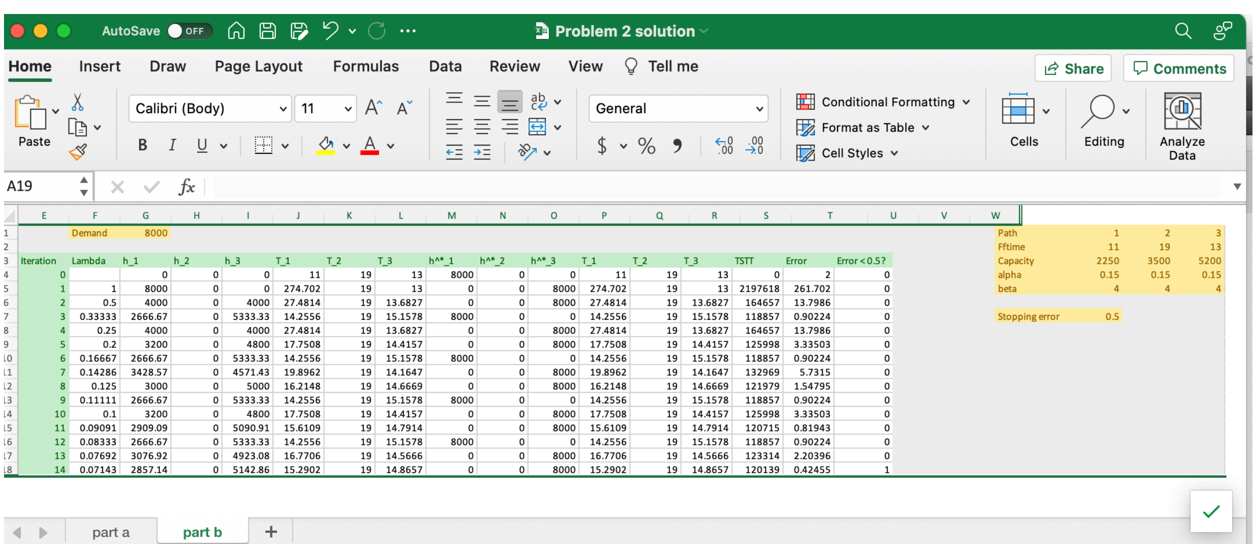
The necessary parameters have been estimated as given below for each path:

Parameter	Name	Value		
		Link 1	Link 2	Link 3
α_a	Calibration constant	0.15	0.15	0.15
$\bar{t}_a$	Free flow travel time	11	19	13
β_a	Calibration constant	4	4	4
Q_a	Saturation flow	2250	3500	5200

Use the method of successive averages to find user equilibrium. (Hint: use Excel. The IF function is useful here.) You will not find the exact user equilibrium. Stop when the maximum difference between path travel times is less than 0.5min.

What is the **flow and travel time** on each link?

Solutions Create an Excel spreadsheet that calculates the method of successive averages:



Demand	8000

Iteration	Lambda	h_1	h_2	h_3	T_1	T_2	T_3	h^*_1	h^*_2	h^*_3	T_1	T_2	T_3	TSTT	Error	Error < 0.5?
0		0	0	0	11	19	13	8000	0	0	11	19	13	0	2	0
1	1	8000	0	0	274.702	19	13	0	0	8000	274.702	19	13	2197618	261.702	0
2	0.5	4000	0	4000	27.4814	19	13.6827	0	0	8000	27.4814	19	13.6827	164657	13.7986	0
3	0.33333	2666.67	0	5333.33	14.2556	19	15.1578	8000	0	0	14.2556	19	15.1578	118857	0.90224	0
4	0.25	4000	0	4000	27.4814	19	13.6827	0	0	8000	27.4814	19	13.6827	164657	13.7986	0
5	0.2	3200	0	4800	17.7508	19	14.4157	0	0	8000	17.7508	19	14.4157	125998	3.33503	0
6	0.16667	2666.67	0	5333.33	14.2556	19	15.1578	8000	0	0	14.2556	19	15.1578	118857	0.90224	0
7	0.14286	3428.57	0	4571.43	19.8962	19	14.1647	0	0	8000	19.8962	19	14.1647	132969	5.7315	0
8	0.125	3000	0	5000	16.2148	19	14.6669	0	0	8000	16.2148	19	14.6669	121979	1.54795	0
9	0.11111	2666.67	0	5333.33	14.2556	19	15.1578	8000	0	0	14.2556	19	15.1578	118857	0.90224	0
10	0.1	3200	0	4800	17.7508	19	14.4157	0	0	8000	17.7508	19	14.4157	125998	3.33503	0
11	0.09091	2909.09	0	5090.91	15.6109	19	14.7914	0	0	8000	15.6109	19	14.7914	120715	0.81943	0
12	0.08333	2666.67	0	5333.33	14.2556	19	15.1578	8000	0	0	14.2556	19	15.1578	118857	0.90224	0
13	0.07692	3076.92	0	4923.08	16.7706	19	14.5666	0	0	8000	16.7706	19	14.5666	123314	2.20396	0
14	0.07143	2857.14	0	5142.86	15.2902	19	14.8657	0	0	8000	15.2902	19	14.8657	120139	0.42455	1

Path	1	2	3
Fftime	11	19	13
Capacity	2250	3500	5200
alpha	0.15	0.15	0.15
beta	4	4	4
Stopping error	0.5		

A.7 Trip distribution with vehicle automation

Consider weekend trips between Plymouth, Golden Valley, and Mendota Heights to three shopping malls: Ridgedale Center, Rosedale Center, and and Mall of America. The current trip table is as follows:

	Ridgedale Center	Rosedale Center	Mall of America
Plymouth	1,174	296	1,689
Golden Valley	526	1,513	2,872
Mendota Heights	271	332	2,690

Due to increasing use of partially automated vehicles (like Tesla), freeway travel times are predicted to improve to the following in 5 years.

	Ridgedale Center	Rosedale Center	Mall of America
Plymouth	11	33	31.4
Golden Valley	33.2	11.5	28.7
Mendota Heights	23.5	17	10.5

Suppose that all else being equal, trips are inversely proportional to travel times.

Assume that all other household and mall characteristics remain constant. In other words, the number of trips made does not change, but their destinations might be different. What is the new trip table?

Solutions First step: calculate productions and attractions. For notational brevity, I will label Plymouth as 1, Golden Valley as 2, Mendota Heights as 3, Ridgedale Center as 4, Rosedale Center as 5, Mall of America as 6.

The productions is the sum of outgoing trips, and attractions is the sum of incoming trips. For example:

$P_1 = 1174+296+1689 = 3159$. $A_4 = 1174+526+271 = 1971$. The productions and attractions are given below:

Zone	Productions	Attractions
Plymouth	3,159	
Shoreview	4,911	
Mendota Heights	3,294	
Minneapolis		1,971
St. Paul		2,142
Mall of America / MSP airport		7,251

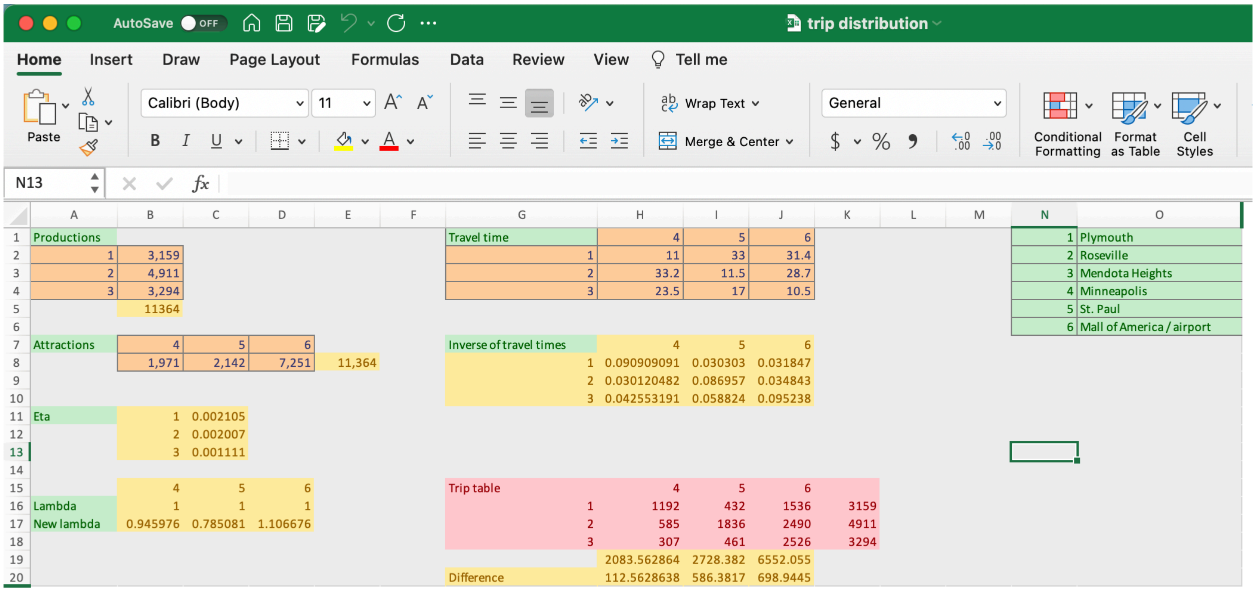
Start with $\lambda_s = 1$ for all destinations.

Next, calculate $\eta_r = \frac{1}{\sum_s \lambda_s A_s \frac{1}{T_{rs}}}$. $\eta_1 = \frac{1}{1\times 1971/11+1\times 2142/33+1\times 7251/31.4} = 0.002105$.

Calculate $d_{rs} = \eta_r \lambda_s P_r A_s \frac{1}{T_{rs}}$. Example: $d_{14} = 0.002105 \times 1 \times 3159 \times 1971/11 = 1192$.

Calculate the new λ value. For example $\lambda_4^* = \frac{1971\times\lambda_4}{1192+585+307} = 0.946$.

Recalculate the trip table with the new lambdas. Repeat several times so that the trips matches attractions.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Productions						Travel time	4	5	6				1	Plymouth
2	1	3,159					1	11	33	31.4				2	Roseville
3	2	4,911					2	33.2	11.5	28.7				3	Mendota Heights
4	3	3,294					3	23.5	17	10.5				4	Minneapolis
5		11364												5	St. Paul
6														6	Mall of America / airport
7	Attractions	4	5	6			Inverse of travel times	4	5	6					
8		1,971	2,142	7,251	11,364		1	0.090909091	0.030303	0.031847					
9							2	0.030120482	0.086957	0.034843					
10							3	0.042553191	0.058824	0.095238					
11	Eta	1	0.002105												
12		2	0.002007												
13		3	0.001111												
14															
15		4	5	6			Trip table	4	5	6					
16	Lambda	1	1	1			1	1192	432	1536	3159				
17	New lambda	0.945976	0.785081	1.106676			2	585	1836	2490	4911				
18							3	307	461	2526	3294				
19								2083.562864	2728.382	6552.055					
20							Difference	112.5628638	586.3817	698.9445					

The final trip table is below:

	Ridgedale Center	Rosedale Center	Mall of America
Plymouth	1,122	333	1,704
Golden Valley	572	1,470	2,869
Mendota Heights	276	340	2,678

A.8 Prisoner's dilemma and user equilibrium route choice slides